EX NAVODAYAN FOUNDATION
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Full Syllabus

JEE-Main

Paper-3

Test Date: **M.M: 300**

TEST INSTRUCTIONS

1. The test is of **3 hours** duration.
2. The test booklet consists of **75 questions**.
3. The maximum marks are **300**.
4. All questions are compulsory.
5. There are three parts in the questions paper consisting of Physics, Chemistry and Mathematics having **25 questions in each part**.

Each Parts Contains –

- 20 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct. All questions are carrying **+4 marks** for right answer and **-1 mark** for wrong answer.
- 05 questions with answer as **numerical value** all questions are carrying **+4 marks** for right answer and **–1 marks** for wrong answers.

Name of the Candidate (in Capital Letter): ______________________

Registration No. ______________________

Invigilator Signature

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Physics

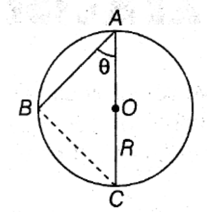
(Single Correct Choice Type)

This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. The dimensions of $\frac{a}{b}$ in the equation $p = \frac{a - t^2}{bx}$ where, p is pressure, x is distance and t is time, are

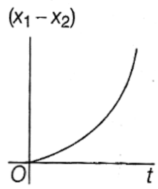
(a) $[M^2LT^{-3}]$ (b) $[MT^{-2}]$ (c) $[ML^3T^{-2}]$ (d) $[LT^{-3}]$

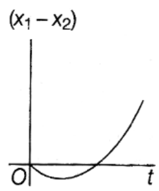
2. A frictionless wire AB is fixed on a sphere of radius R and A very small spherical ball slips on this wire. The time taken by this ball to slip from A to B is

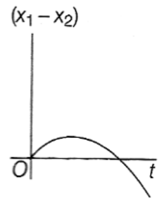


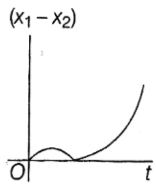
(a) $\frac{2\sqrt{gR}}{g\cos\theta}$ (b) $2\sqrt{gR}\frac{\cos\theta}{g}$ (c) $2\sqrt{\frac{R}{g}}$ (d) $\frac{gR}{\sqrt{g\cos\theta}}$

3. A body is at rest at x = 0. At t = 0, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through x = 0 moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time t?

(a) 

(b) 

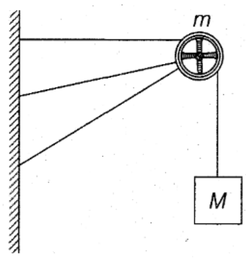
(c) 

(d) 

4. A water fountain on the ground sprinkless water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is

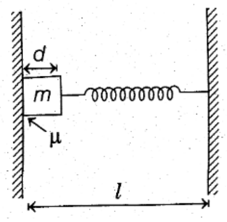
(a) $\pi\frac{v^4}{g^2}$ (b) $\frac{\pi}{2}\frac{v^4}{g^2}$ (c) $\pi\frac{v^2}{g^2}$ (d) $\pi\frac{v^2}{g}$

5. A string of negligible mass, going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given by



(a) $\sqrt{2}Mg$ (b) $\sqrt{[(M+m)^2 + m^2]}g$ (c) $2Mg$ (d) $\sqrt{[(M+m)+m]^2}g$

6. A block of mass is pressed against a vertical surface by a spring of unstretched length l. If the coefficient of friction between the block and the surface is μ, then choose the correct statement.



(a) If spring constant $k = \frac{2mg}{\mu d}$, block will not be in equilibrium

(b) Minimum spring constant k_{min} to keep the block of mass m in equilibrium is $\frac{mg}{\mu d}$

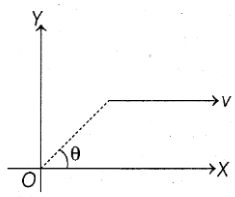
(c) If spring constant is $k = \frac{2mg}{\mu d}$, the normal reaction is $\frac{mg}{\mu}$

(d) In the part (c), force of friction is 2mg

7. When a rubber band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$, where, a and b are constants. The work done in stretching the unstretched rubber band by L is

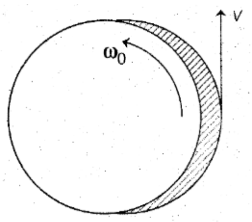
(a) $aL^2 + bL^3$ (b) $\frac{1}{2}(aL^2 + bL^3)$ (c) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (d) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

8. A particle moves parallel to X-axis with constant velocity v as shown in the figure. The angular velocity of the particle about the origin O is



(a) remains constant (b) continuously increasing
(c) continuously decreasing (d) oscillates

9. A child with mass m is standing at the edge of a merry-go-round having moment of inertia l, radius R and initial angular velocity ω_0 as shown in the figure.

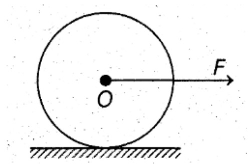


The child jumps off the edge of the merry-go-round with tangential velocity v w.r.t. ground. The new angular velocity of the merry-go-round is

(a) $\left(\frac{l\omega_0^2 - mv^2}{l}\right)^{\frac{1}{2}}$ (b) $\left(\frac{(l + mR^2)\omega_0^2 - mv^2}{l}\right)^{\frac{1}{2}}$

(c) $\left(\frac{l\omega_0 - mvR}{l}\right)$ (d) $\left[\frac{(l + mR^2)\omega_0 - mvR}{l}\right]$

10. A horizontal force F acts on the sphere at its centre as shown. Coefficient of friction between ground and sphere is μ. What is the maximum value of F, for which there is no slipping?

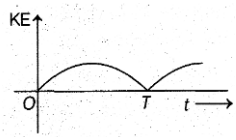


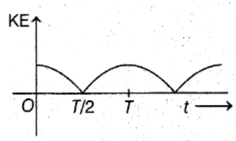
(a) $F \le \frac{5}{2}\mu mg$ (b) $F \le \frac{7}{2}\mu mg$ (c) $F \le \frac{9}{2}\mu mg$ (d) $F \le \frac{3}{2}\mu mg$

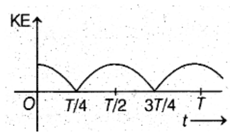
11. Two particles of equal mass m go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is

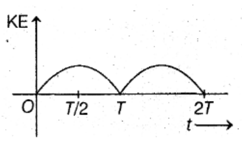
(a) $\sqrt{\frac{Gm}{R}}$ (b) $\sqrt{\frac{Gm}{4R}}$ (c) $\sqrt{\frac{Gm}{3R}}$ (d) $\sqrt{\frac{Gm}{2R}}$

12. A particle is executing simple harmonic motion with a time period T. At time t = 0, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look, like

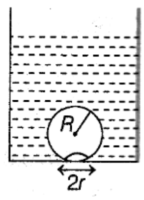
(a) 

(b) 

(c) 

(d) 

13. On heating water, bubbles being formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If $r << R$ and the surface tension of water is T, value of r just before bubbles detach is (density of water is ρ)



(a) $R^2\sqrt{\frac{2\rho_w g}{3T}}$ (b) $R^2\sqrt{\frac{\rho_w g}{6T}}$ (c) $R^2\sqrt{\frac{\rho_w g}{T}}$ (d) $R^2\sqrt{\frac{3\rho_w g}{T}}$

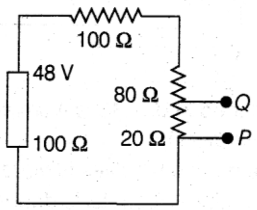
14. A Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then, T_1 and T_2 are respectively

(a) 372 K and 330 K (b) 330 K and 268 K (c) 310 K and 248 K (d) 372 K and 310 K

15. If the electric flux entering and leaving an enclosed surface respectively are ϕ_1 and ϕ_2, the electric charge inside the surface will be

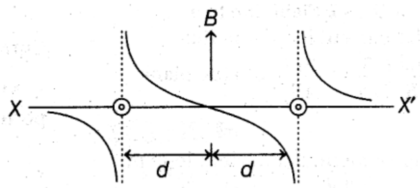
(a) $\frac{\phi_2 - \phi_1}{\varepsilon_0}$ (b) $\frac{\phi_1 + \phi_2}{\varepsilon_0}$ (c) $\frac{\phi_1 - \phi_2}{\varepsilon_0}$ (d) $\varepsilon_0(\phi_1 + \phi_2)$

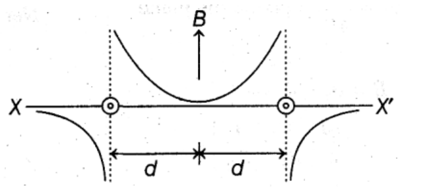
16. In the circuit in the figure below, the potential difference across P and Q will be nearest to

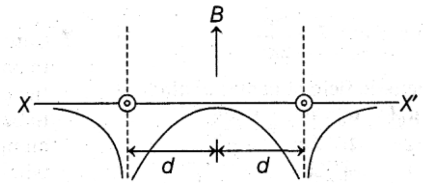


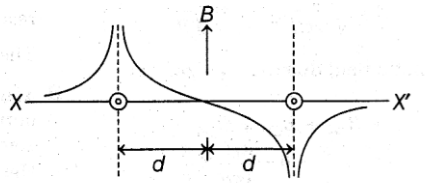
(a) 9.6 V (b) 6.6 V (c) 4 V (d) 3.2 V

17. Two long parallel wires are at a distance 2d apart. They carry steady equal current flowing out of the plane of the paper as shown. The variation of the magnetic field along the line XX' is given by

(a)


(b)


(c)


(d)


18. A cylindrical bar magnet is rotated about its axis. A wire is connected from the axis and is made to touch the cylindrical surface through a contact. Then,

(a) a direct current flows in the ammeter A

(b) no current flows through the ammeter A

(c) an alternating sinusoidal current flows through the ammeter A with a time period $T = \frac{2\pi}{\omega}$

(d) a time varying non-sinusoidal current flows through the ammeter A

19. A fish looking up through the water sees the outside world, contained in a circular horizon. If the refractive index of water is $\frac{4}{3}$ and the fish is 12 cm below the water surface, the radius of this circle in cm, is

(a) $36\sqrt{7}$ (b) $\frac{36}{\sqrt{7}}$ (c) $36\sqrt{5}$ (d) $4\sqrt{5}$

20. When an unpolarised light of intensity l_0 is incident on a polarising sheet, the intensity of the light which does not get transmitted is

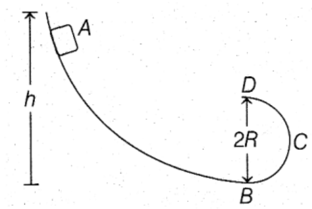
(a) $\frac{1}{2}l_0$ (b) $\frac{1}{4}l_0$ (c) zero (d) l_0

(Integer Type Questions)

This Section contains **05 Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

21. A body starts from the origin and moves along the axis such that the velocity at any instant is given by $v = 4t^3 - 2t$, where t is in second and the velocity in ms^{-1}. Find the acceleration of the particle when it is at a distance of 2 m from the origin.

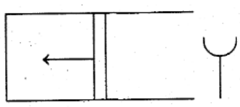
22. A frictionless track ABCD ends in a semi-circular loop of radius R. A body slides down the track from point A which is at a height h = 5 cm. Maximum value of R for the body to successfully complete the loop is (cm)



23. Two particles execute simple harmonic motion on same straight line with same mean position, same time period 6 s and same amplitude 5 cm. Both the particles start SHM from their mean position (in same direction) with a time gap of 1 s. Find the maximum separation between the two particles during their motion in (cm)

24. Two simple pendulums of length 1 m and 4 m respectively are both given small displacement in the same direction. The shorter pendulum has completed number of oscillations equal to

25. A piston fitted in cylindrical pipe is pulled as shown in the figure. A tuning fork is sounded at open end and loudest sound is heard at open length 13 cm, 41 cm and 69 cm. The frequency of tuning fork if velocity of sound is $350\,ms^{-1}$, is



Chemistry

(Single Correct Choice Type)

This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. Consider oxidation of the following alcohols

I. $CH_2=CHCH_2OH \xrightarrow{A} CH_2=CHCHO$

II. $(CH_3)_3COH \xrightarrow{B} CH_3\underset{\substack{|\\CH_3}}{C}=CH_2$

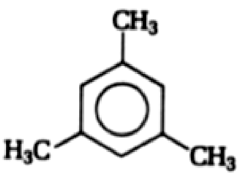
III. $CH_3CH_2OH \xrightarrow{C} CH_3CO_2H$

IV. $CH_3CH=CHCH_3 \xrightarrow{D} CH_3CO_2H$

A, B, C and D are oxidizing agents which are respectively

	A	B	C	D
(a)	MnO_2	Cu/Δ	H_2CrO_4	$KMnO_4$/Δ
(b)	Cu/Δ	MnO_2	H_2CrO_4	$KMnO_4$/Δ
(c)	MnO_2	Cu/Δ	$KMnO_4$/Δ	H_2CrO_4
(d)	MnO_2	H_2CrO_4	Cu/Δ	$KMnO_4$

2. $CH_3CN \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } CH_3MgBr(2mol)} A \xrightarrow{conc.H_2SO_4} B$; A and B are

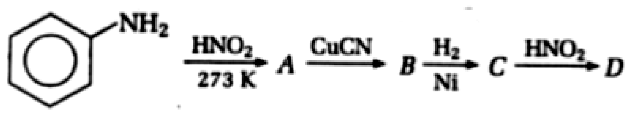
(a) CH_3COCH_3, 

(b) $(CH_3)_3COH$, $(CH_3)_2C=CH_2$

(c) $(CH_3)_2CHOH$, $CH_3CH=CH_2$

(d) None of the above is correct

3. Aniline in a set of reactions yielded a product D.



The structure of the product D would be

(a) $C_6H_5CH_2NH_2$ (b) $C_6H_5NHCH_2CH_3$ (c) C_6H_5NHOH (d) $C_6H_5CH_2OH$

4. The elements with atomic number 117 and 120 are yet to be discovered. In which group would you place these elements when discovered?

(a) 17, 2 (b) 16, 4 (c) 15, 3 (d) 18, 2

5. The correct statement regarding SO_2 molecule is:

(a) Two pπ – dπ bonds
(b) Molecules has 2 lone pair, 2σ bonds and 2π bonds
(c) two pπ – pπ bonds
(d) one pπ – pπ and one pπ – dπ bond

6. Which of the following is incorrectly match?

	Hybridisation	Geometry	Orbitals use
(a)	sp^3d	Trigonal bipyramidal	$s + p_x + p_y + p_z + d_{z^2}$
(b)	sp^3d^3	Pentagonal bipyramidal	$s + p_x + p_y + p_z + d_{x^2-y^2} + d_{z^2} + d_{xy}$
(c)	sp^3d^2	Capped ocetahedral	$s + p_x + p_y + p_z + d_{x^2-y^2} + d_{z^2}$
(d)	sp^3	Tetrahedral	$s + p_x + p_y + p_z$

7. The ground state electronic configurations of the elements, U, V, W, X and Y (these symbols do not have any chemical significance) are as follows:

U $1s^2 2s^2 2p^3$

V $1s^2 2s^2 2p^6 3s^1$

W $1s^2 2s^2 2p^6 3s^2 3p^2$

X $1s^2 2s^2 2p^6 3s^2 3p^6 3d^5 4s^2$

Y $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6$

Determine which sequence of elements satisfy the following statements:

(i) Element forms a carbonate which is not decomposed on heating

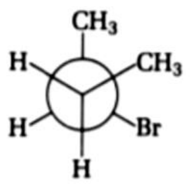
(ii) Element is most likely to form coloured ionic compounds

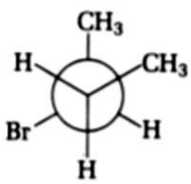
(iii) Element has largest atomic radius

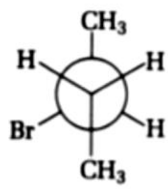
(iv) Element forms only acidic oxide

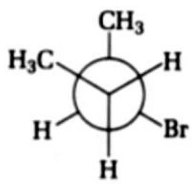
(a) V W Y U (b) V X Y W (c) V W Y X (d) V X W U

8. In the dehydrohalogenation of 2-bromobutane, which conformation leads to the formation of cis-2-butene?

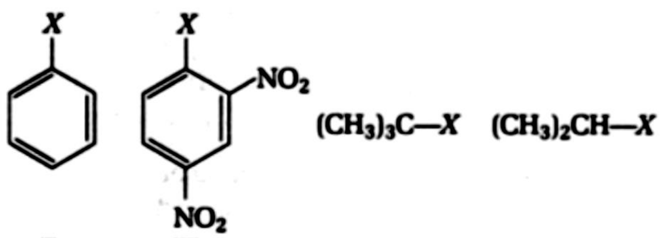
(a) 

(b) 

(c) 

(d) 

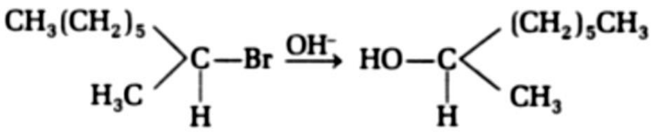
9. The correct order of increasing reactivity of C—X bond towards nucleophile in the following compound is



(I) (II) (III) (IV)

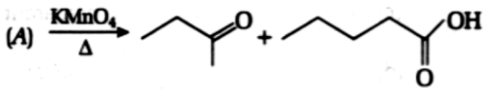
(a) III < II < I < IV (b) I < II < IV < III (c) II < III < I < IV (d) IV < III < I < II

10. The following reaction is described as

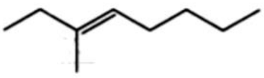


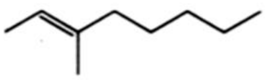
(a) S_{E^2} (b) S_{N^2} (c) S_{N^1} (d) S_{N^0}

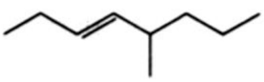
11. Alkene

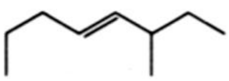


A is

(a) 

(b) 

(c) 

(d) 

12. Consider the following complex : $[Co(NH_3)_5CO_3]ClO_4$

The coordination number, oxidation number, no. of d-electron and number of unpaired d-electron on the metal are respectively:

(a) 6, 2, 7, 3 (b) 7, 2, 7, 1 (c) 5, 3, 6, 4 (d) 6, 3, 6, 0

13. Amongst the following compounds

(I) $H_5P_3O_{10}$ (II) $H_6P_4O_{13}$ (III) $H_5P_5O_{15}$ (IV) $H_7P_5O_{16}$

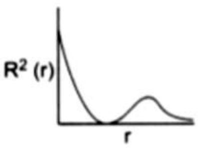
Non-cyclic phosphates are:

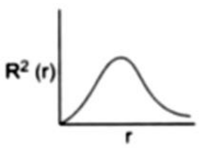
(a) I, III (b) I, II, III (c) I, II, IV (d) I, II, III, IV

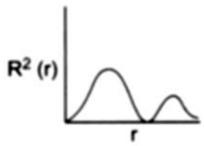
14. Calculate number of electron present in 9.5 g of PO_4^{3-} :

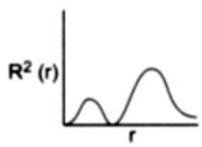
(a) 6 (b) 5 N_A (c) 0.1 N_A (d) 4.7 N_A

15. The variation of radial probability density $R^2(r)$ as a function of distance r of the electron from the nucleus for 3 p-orbital:

(a) 

(b) 

(c) 

(d) 

16. The cell reaction $2Ag^+(aq) + H_2(g) \longrightarrow 2H^+(aq) + 2Ag(s)$, is best represented by:

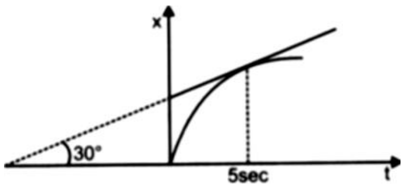
(a) Ag(s) | Ag^+(aq) | | H^+(aq) | H_2(g) | Pt(s) (b) Pt(s) | H_2(g) | H^+(aq) | | Ag^+(aq) | Ag(s)

(c) Ag(s) | Ag^+(aq) | | H_2(g) | H^+(aq) | Pt(s) (d) Ag^+(aq) | Ag(s) | | H_2(g) | H^+(aq)

17. For a first order reaction

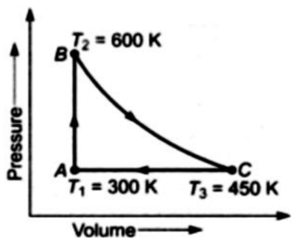
$$A_{(g)} \longrightarrow B_{(g)} + C_{(g)}$$
$$t = 0 \Rightarrow a$$
$$t = t \Rightarrow (a - x)$$

If concentration of A is 0.1 M at 5 sec. then from following graph the rate constant is:



(a) 10.77 s^{-1} (b) 5.77 s^{-1} (c) 2.3 s^{-1} (d) 1 s^{-1}

18. A heat engine carries one mole of an ideal mono-atomic gas around the cycle as shown in the figure. Select the correct option:



(a) q_{AB} = 450 R and q_{CA} = –450 R (b) q_{AB} = 450 R and q_{CA} = –225 R
(c) q_{AB} = 450 R and q_{CA} = –375 R (d) q_{AB} = 375 R and q_{CA} = –450 R

19. For the dissociation reaction $N_2O_4(g)$ $2NO_2(g)$, the degree of dissociation (α) in terms of K_p and total equilibrium pressure P is:

(a) $\alpha = \sqrt{\frac{4P + K_P}{K_P}}$ (b) $\alpha = \sqrt{\frac{K_P}{4P + K_P}}$ (c) $\alpha = \sqrt{\frac{K_P}{4P}}$ (d) None of these

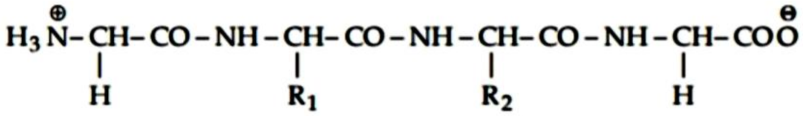
20. Two liquids A and B have vapour pressure in the ratio $P_A^o : P_B^o = 1:3$ at a certain temperature. Assume A and B form an ideal solution and the ratio of mole fraction of A to B in the vapour phase is 4 : 3. Then the mole fraction of B in the solution at the same temperature is:

(a) $\frac{1}{5}$ (b) $\frac{2}{3}$ (c) $\frac{4}{5}$ (d) $\frac{1}{4}$

(Integer Type Questions)

This Section contains **10 Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

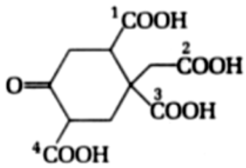
21. The substituents R_1 and R_2 for nine peptides are listed in the table given below. How many of these peptides are negatively charged at pH = 7.0?



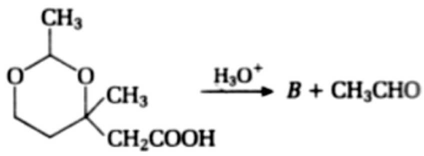
Peptide	R_1	R_2
I	H	H
II	H	CH_3
III	CH_2COOH	H
IV	CH_2CONH_2	$(CH_2)_4NH_2$
V	CH_2CONH_2	CH_2CONH_2
VI	$(CH_2)_4NH_2$	$(CH_2)_4NH_2$
VII	CH_2COOH	CH_2CONH_2
VIII	CH_2OH	$(CH_2)_4NH_2$
IX	$(CH_2)_4NH_2$	CH_3

22. The dissociation constant of a substituted benzoic acid at 25°C is 1.0×10^{-4}. The pH of 0.01 M solution of its sodium salt is __________

23. Which $-COOH$ is lost due to heating?



24. On standing in dilute aqueous acid A change to B



How many O-atoms are present in B?

25. How much mL water should be added to 1 mL of 1M $KMnO_4$ solution so that it is 0.5 N in a titration in acidic medium?

Mathematics

(Single Correct Choice Type)

This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. Let L_1 be a straight line passing through the origin and L_2 be the straight line $x+y=1$. If the intercepts made by the circle $x^2+y^2-x+3y=0$ on L_1 and L_2 are equal, then which of the following equation can represent L_1?

(a) $x+7y=0$ (b) $x-y=0$ (c) $x-7y\ 0$ (d) Both (a) and(b)

2. Let α_1,α_2 and β_1,β_2 be the roots of $ax^2+bx+c=0$ and $px^2+qx+r=0$ respectively. If the system of equations $\alpha_1 y+\alpha_2 z=0$ and $\beta_1 y+\beta_2 z=0$ has a non-trivial solution, then

(a) $\frac{b^2}{q^2}=\frac{ac}{pr}$ (b) $\frac{c^2}{r^2}=\frac{ab}{pq}$ (c) $\frac{a^2}{p^2}=\frac{bc}{qr}$ (d) None of these

3. The equation $\sin x+x\cos x=0$ has at least one root in

(a) $\left(-\frac{\pi}{2},0\right)$ (b) $(0,\pi)$ (c) $\left(\pi,\frac{3\pi}{2}\right)$ (d) $\left(0,\frac{\pi}{2}\right)$

4. The area above the x-axis enclose by the curves $x^2-y^2=0$ and $x^2+y-2=0$ is

(a) $\frac{5}{3}$ (b) $\frac{7}{3}$ (c) $\frac{8}{3}$ (d) $\frac{10}{3}$

5. If the function $f:[0,16]\to R$ is differentiable. If $0<\alpha<1$ and $1<\beta<2$, then $\int_0^{16} f(t)dt$ is equal to.

(a) $4[\alpha^3 f(\alpha^4)-\beta^3 f(\beta^4)]$ (b) $4[\alpha^3 f(\alpha^4)+\beta^3 f(\beta^4)]$

(c) $4[\alpha^4 f(\alpha^3)+\beta^4 f(\beta^3)]$ (d) $4[\alpha^2 f(\alpha^2)+\beta^2 f(\beta^2)]$

6. Three distinct points $P(3u^2,2u^3),Q(3v^2,2v^3)$ and $R(3w^2,2w^3)$ are collinear then –

(a) $uv+vw+wu=0$ (b) $uv+vw+wu=3$ (c) $uv+vw+wu=2$ (d) $uv+ww+wu=1$

7. Let 'a' denote the roots of equation $\cos(\cos^{-1}x)+\sin^{-1}\sin\left(\frac{1+x^2}{2}\right)=2\sec^{-1}(\sec x)$ then possible values of $[|10a|]$ where [.] denotes the greatest integer function will be

(a) 1 (b) 5 (c) 10 (d) Both (a) and (c)

8. If $\int \frac{dx}{x+x^7}=p(x)$ then, $\int \frac{x^6}{x+x^7}dx$ is equal to:

(a) $\ln|x|-p(x)+c$ (b) $\ln|x|+p(x)+c$ (c) $x-p(x)+c$ (d) $x+p(x)+c$

9. The value of the definite integral $\int_0^{3\pi/4}[(1+x)\sin x+(1-x)\cos x]dx$ is –

(a) $2\tan\frac{3\pi}{8}$ (b) $2\tan\frac{\pi}{4}$ (c) $2\tan\frac{\pi}{8}$ (d) 0

10. Area of triangle formed by common tangents to the $x^2+y^2-6x=0$ and $x^2+y^2+2x=0$ is –

(a) $3\sqrt{3}$ (b) $2\sqrt{3}$ (c) $9\sqrt{3}$ (d) $6\sqrt{3}$

11. The locus of the centres of the circles which cut the circles $x^2+y^2+4x-6y+9=0$ and $x^2+y^2-5x+4y-2=0$ orthogonally is -

(a) $9x+10y-7=0$ (b) $x-y+2=0$ (c) $9x-10y+11=0$ (d) $9x+10y+7=0$

12. The straight line joining any point P on the parabola $y^2=4ax$ to the vertex and perpendicular from the focus to the tangent at P, intersect at R, then the equation of the locus of R is-

(a) $x^2+2y^2-ax=0$ (b) $2x^2+y^2-2ax=0$ (c) $2x^2+2y^2-ay=0$ (d) $2x^2+y^2-2ay=0$

13. If $z+1/z=2\cos\theta$, then the value of $|(z^{2n}-1)/(z^{2n}+1)|$

(a) $|\tan n\theta|$ (b) $\tan n\theta$ (c) $|\cot n\theta|$ (d) $\cot n\theta$

14. The two curves $x^3-3xy^2+2=0$ and $3x^2y-y^3=2$

(a) Cuts at right angle (b) touch each other

(c) cut at an angle $\frac{\pi}{3}$ (d) cut at an angle $\frac{\pi}{4}$

15. Period of $\frac{\sin\theta+\sin 2\theta}{\cos\theta+\cos 2\theta}$ is

(a) 2π (b) π (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{3}$

16. If $1,\omega,\omega^2,......\omega^{n-1}$ are the n roots of unity , then $(1-\omega)(1-\omega^2).....(1-\omega^{n-1})$ equals

(a) 0 (b) 2 (c) n (d) n^2

17. If s,s' are the length of the perpendicular on a tangent from the foci, a , a' are those from the vertices is that from the centre and e is the eccentricity of the ellipse , $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $\frac{ss'-c^2}{aa'-c^2}=$

(a) e (b) $1/e$ (c) $1/e^2$ (d) e^2

18. One percent of the population suffers from a certain disease. There is blood test for this diesease, and it is 99% accurate, in other words, the probability that it gives the correct answer is 0.99, regardless of whether the person is sick or healthy. A person takes the blood test, and the result says that he has the disease. The probability that he actually has the disease, is –

(a) 0.99% (b) 25% (c) 50% (d) 75%

19. Set of values of m for which two points P and Q lie on the line $y=mx+8$ so that $\angle APB=\angle AQB=\frac{\pi}{2}$ where $A\equiv(-4,0), B\equiv(4,0)$ is –

(a) $(-\infty,-\sqrt{3})\cup(\sqrt{3},\infty)-\{-2,2\}$ (b) $[-\sqrt{3},-\sqrt{3}]-\{-2,2\}$

(c) $(-\infty,-1)\cup(1,\infty)-\{-2,2\}$ (d) $\{-\sqrt{3},\sqrt{3}\}$

20. The trace $T_r(A)$ of a 3×3 matrix $A=(a_{ij})$ is defined by the relation $T_r(A)=a_{11}+a_{22}+a_{33}$ (i.e, $T_r(A)$ is sum of the main diagonal elements). Which of the following statements cannot hold?

(a) $T_r(kA)=kT_r(A)$ (k is a scalar) (b) $T_r(A+B)=T_r(A)+T_r(B)$

(c) $T_r(I_3)=3$ (d) $T_r(A^2)=T_r(A)^2$

(Integer Type Questions)

This Section contains **10 Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

21. If $x^2-2x\cos\theta+1=0$, then the value of $x^{2n}-2x^n\cos n\theta+1, n\in N$ is equal to –

22. Given $\vec{A}=2\hat{i}+3\hat{j}+6\hat{k}, \vec{B}=\hat{i}+\hat{j}-2\hat{k}$ and $\vec{C}=\hat{i}+2\hat{j}+\hat{k}$ compute the value of $|\vec{A}\times[\vec{A}\times(\vec{A}\times\vec{B})].\vec{C}|$.

23. The sum to infinity of the series $\frac{1}{1}+\frac{1}{1+2}+\frac{1}{1+2+3}+\frac{1}{1+2+3+4}+.......$ is equal to –

24. Let $a_n=\int_0^{\pi/2}(1-\sin t)^n\sin 2t\,dt$ then $\lim_{n\to\infty}\sum_1^n\frac{a_n}{n}$ is equal to

25. The distance of the point $(-1,-5,-10)$ from the point of intersection of the line, $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane, $x-y+z=5$, is

Answer – key

Physics
1. b
2. c
3. b
4. a
5. b
6. b
7. c
8. c
9. d
10. b
11. b
12. c
13. a
14. d
15. d
16. d
17. a
18. b
19. b
20. a
21. 22
22. 8
23. 4
24. 3
25. 9

Chemistry
1. A
2. B
3. D
4. A
5. D
6. C
7. B
8. A
9. B
10. B
11. A
12. D
13. C
14. B
15. D
16. B
17. B
18. C
19. B
20. A
21. 4
22. 8
23. 4
24. 3
25. 9

Math
1. d
2. a
3. b
4. b
5. b
6. a
7. d
8. a
9. a
10. a
11. c
12. b
13. b
14. a
15. c
16. c
17. d
18. c
19. a
20. d

Integer
21. 0
22. 343
23. 2
24. 0.5
25. 13

SOLUTION

Physics

1. (b)

$$p = \frac{a - t^2}{bx}$$

$\Rightarrow \quad pbx = a - t^2$

$\Rightarrow \quad [pbx] = [a] = [T^2]$

or $[b] = \frac{[T^2]}{[p][x]} = \frac{[T^2]}{[ML^{-1}T^{-2}][L]} = [M^{-1}T^4]$

$\therefore \left[\frac{a}{b}\right] = \frac{[T^2]}{[M^{-1}T^4]} = [MT^{-2}]$

2. (c)

Acceleration of the body down the plane = $g \cos\theta$

Distance travelled by ball in time t second is

$$AB = \frac{1}{2}(g\cos\theta)t^2 \quad \text{...(i)}$$

From Δ ABC,

$$AB = 2R\cos\theta \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$2R\cos\theta = \frac{1}{2}g\cos\theta t^2$$

$$t^2 = \frac{4R}{g}$$

or $$t = 2\sqrt{\frac{R}{g}}$$

3. (b)

As, $x_1(t) = \frac{1}{2}at^2$ and $x_2(t) = vt$

$\therefore \quad x_1 - x_2 = \frac{1}{2}at^2 - vt \quad$ (parabola)

Clearly, graph (b) represents it correctly.

4. (a)

Maximum range of water coming out of the fountain,

$$R_m = \frac{v^2}{g}$$

$\therefore$ Total area around fountain,

$$A = \pi R_m^2 = \pi\frac{v^4}{g^2}$$

5. (b)

Force on the pulley, by the clamp

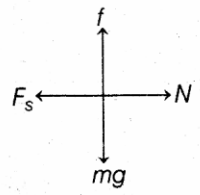
= Resultant of forces $(M + m)g$ acting along horizontally and mg acting vertically downwards

$= \sqrt{(Mg + mg)^2 + (mg)^2}$

$= \sqrt{[(M + m)^2 + m^2]}g$

6. (b)

Free body diagram of the block is



Here, $N = F_s = kd$

and $mg = f \le \mu N = \mu kd$

or $k \ge \frac{mg}{\mu d}$

7. (c)

Work Done by variable force

$dw = \vec{F}\cdot d\vec{x}$

$= \int_0^L (ax + bx^2)\,dx$

$= \frac{aL^2}{2} + \frac{bL^3}{3}$

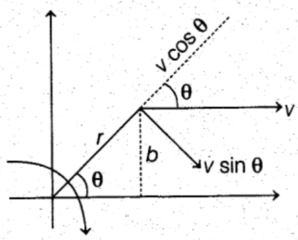
8. (c)

Since, $\omega = \frac{v}{r}$

But, here

$v = v\sin\theta$

and $\quad r = b/\sin\theta$



$\therefore \quad \omega = \frac{v\sin\theta}{b/\sin\theta}$

$= \frac{v\sin^2\theta}{b}$

or $\quad \omega \propto \sin^2\theta$

[as v and b are constants]

$\therefore$ When particle moves further parallel to the X-axis, θ decreases result angular velocity decreases.

Hence, (c) is the correct option.

9. (d)

Since, in this condition,
Initial angular momentum
= Final angular momentum

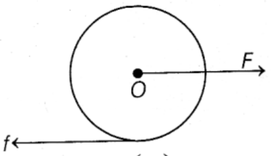
$\therefore (I + mR^2)\omega_0 = (mvR) + I\omega'$

or $\omega' = \dfrac{(I + mR^2)\omega_0 - mvR}{I}$

Hence, (d) is the correct option.

10. (b)

Here, $F - f = ma$ ... (i)



and $\tau = I\alpha = I\left(\dfrac{a}{R}\right)$

$\Rightarrow \quad f.R = \dfrac{2}{5}mR^2 \times \dfrac{a}{R}$

or $\quad f = \dfrac{2}{5}ma$ ...(ii)

From Eqs. (i) and (ii), we get

$f = \dfrac{2}{7}F$

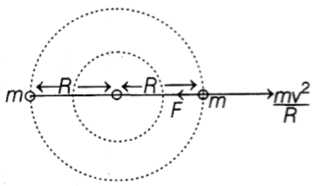
or $\quad \dfrac{2}{7}F \le \mu mg$

or $\quad F \le \dfrac{7}{2}\mu mg$

Hence, (b) is the correct option.

11. (b)

As gravitational force provides necessary centripetal force.



i.e. $F = \dfrac{Gm^2}{(2R)^2} = \dfrac{mv^2}{R}$

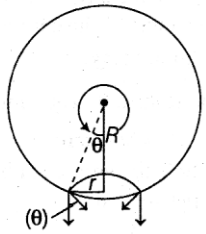
$\Rightarrow v = \sqrt{\dfrac{Gm}{4R}}$

12. (c)

KE is maximum at mean position and minimum at extreme position $\left(\text{at } t = \dfrac{T}{4}\right)$.

13. (a)

The bubble will detach if, Buoyant force ≥ Surface tension force



$\int \sin\theta\, T \times dl = T(2\pi r)\sin\theta$

$\dfrac{4}{3}\pi R^3 \rho_w g \ge \int T \times dl \sin\theta$

$(\rho_w)\left(\dfrac{4}{3}\pi R^3\right)g \ge (T)(2\pi r)\sin\theta$

As, $\quad \sin\theta = \dfrac{r}{R}$

Solving, $r = \sqrt{\dfrac{2\rho_w R^4 g}{3T}} = R^2\sqrt{\dfrac{2\rho_w g}{3T}}$

14. (d)

$\because \eta_1 = 1 - \dfrac{T_2}{T_1}$

$\Rightarrow \quad \dfrac{1}{6} = 1 - \dfrac{T_2}{T_1}$

$\Rightarrow \quad \dfrac{T_2}{T_1} = \dfrac{5}{6}$ ...(i)

$\therefore \quad \eta_2 = 1 - \dfrac{T_2 - 62}{T_1}$

$\Rightarrow \quad \dfrac{1}{3} = 1 - \dfrac{T_2 - 62}{T_1}$ ...(ii)

On solving Eqs. (i) and (ii), we get

$T_1 = 372$ K and $T_2 = 310$ K

15. (d)

Let $-q_1$ be the charge, due to which flux ϕ_1 is entering the surface.

$\phi_1 = \dfrac{-q_1}{\varepsilon_0}$ or $q_1 = -\phi_1\varepsilon_0$

Let $+q_2$ be the charge, due to which flux ϕ_2, is leaving the surface.

$\phi_2 = \dfrac{q_2}{\varepsilon_0}$ or $q_2 = \varepsilon_0\phi_2$

Electric charge inside the surface

$= q_2 - q_1 = \varepsilon_0\phi_2 + \varepsilon_0\phi_1 = \varepsilon_0(\phi_2 + \phi_1)$

16. (d)

Total resistance of circuit
$= 100 + 100 + 80 + 20 = 300\,\Omega$

Current $I = \dfrac{48}{300} = 0.16$ A

Potential difference across P and $Q = 20 \times 0.16 = 3.2$ V

17. (a)

The magnetic field in between because of each will be in opposite direction

$$B_{\text{in between}} = \frac{\mu_0 I}{2\pi x}\hat{j} - \frac{\mu_0 I}{2\pi(2d-x)}(-\hat{j})$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{x} - \frac{1}{2d-x}\right](\hat{j})$$

at $x = d$, $B_{\text{in between}} = 0$

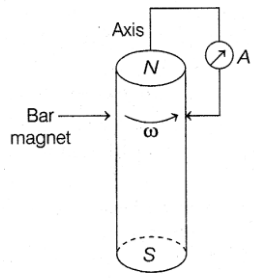
For $x < d$, $B_{\text{in between}} = (\hat{j})$ and

For $x > d$, $B_{\text{in between}} = (-\hat{j})$

Towards x, net magnetic field will add up and direction will be $(-\mathbf{j})$ and towards x', net magnetic field will add up and direction will be $(\mathbf{j})$.

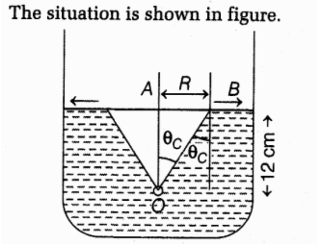
18. (b)

When cylindrical bar magnet is rotated about its axis,no change in flux linked with the circuit takes place, consequently no emf induces and hence,no current flows through the ammeter A.



19. (b)

The situation is shown in figure.



$$\sin\theta_C = \frac{1}{\mu}$$

$$\tan\theta_C = \frac{AB}{OA}$$

$\therefore$ $AB = OA\tan\theta_C$

or $AB = \dfrac{OA}{\sqrt{n^2-1}} = \dfrac{12}{\sqrt{\left(\frac{4}{3}\right)^2 - 1}} = \dfrac{36}{\sqrt{7}}$

20. (a)

$$I = I_0\cos^2\theta$$

Intensity of polarised light $= \dfrac{I_0}{2}$

$\therefore$ Intensity of untransmitted light

$$= I_0 - \frac{I_0}{2} = \frac{I_0}{2}$$

21. (22)

$$v = 4t^3 - 2t \qquad \text{...(i)}$$

$$\frac{dx}{dt} = 4t^3 - 2t$$

On integration, we get,

$$x = 2 = t^4 - t^2$$

Let $t^2 = \alpha$

$\therefore$ $2 = \alpha^2 - \alpha$ ...(ii)

Let $t^2 = \alpha$

$$\alpha^2 - \alpha - 2 = 0$$

$$(\alpha - 2)(\alpha + 1) = 0$$

$\therefore$ $\alpha = 2,\ \alpha = -1,$

which is not possible

$$t^2 = \alpha = 2 \text{ or } t = \sqrt{2},$$

Differentiating Eq. (i) w.r.t. t,

$$\frac{dv}{dt} = 12t^2 - 2$$

$$a = 12 \times 2 - 2 = 22\ \text{ms}^{-2}$$

22. (2)

Velocity at the bottom is $\sqrt{2gh}$

For completing the loop,

$$\sqrt{2gh} = \sqrt{5gR}$$

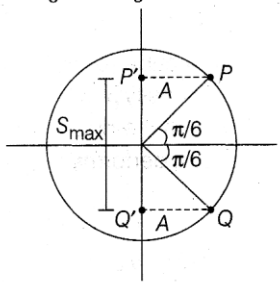
Hence, $R = 2h/5$

$= (2 \times 5)/5 = 2$ cm

23. (5)

Phase difference,

$$\phi = \omega t = \frac{2\pi}{6} \times 1 = \frac{\pi}{3}\ \text{rad}$$



The maximum separation between the two particles is

$$S_{max} = 2A\sin\frac{\pi}{6}$$

or $S_{max} = 2 \times 5 \times \dfrac{1}{2} = 5\text{cm}$

24. (2)

Let T_1 and T_2 be the time period of shorter length and larger length pendulums respectively. According to question,

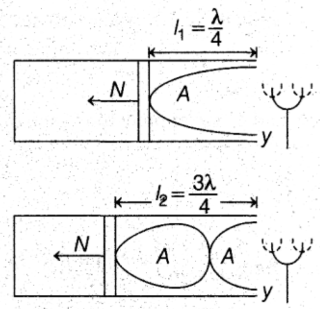
$$nT_1 = (n-1)T_2$$

So, $n\,2\pi\sqrt{\dfrac{1}{8}} = (n-1)2\pi\sqrt{\dfrac{4}{8}}$

or $n = (n-1)2 = 2n - 2 \Rightarrow n = 2$

25. (625)

In a closed organ pipe in which length of air-column can be increased or decreased, the first resonance occurs at $\lambda/4$ and second resonance occurs at $3\lambda/4$.



Thus, at first resonance,

$$\frac{\lambda}{4} = 13 \quad \text{...(i)}$$

and at second resonance,

$$\frac{3\lambda}{4} = 41 \quad \text{...(ii)}$$

Subtracting Eq. (i) from Eq. (ii), we have

$$\frac{3\lambda}{4} - \frac{\lambda}{4} = 41 - 13$$

$\therefore$ $\lambda = 56$ cm

Hence, frequency of tuning fork,

$$n = \frac{v}{\lambda} = \frac{350}{56 \times 10^{-2}} = 625\text{Hz}$$

Chemistry

(1) A $-CH_2-OH$ (1°) $\xrightarrow{MnO_2}$ CHO

3° Alc. $\xrightarrow{Cu, -H_2O}$ ene

1° Alc. $\xrightarrow{H_2CrO_4}$ $-COOH$

Alkene $\xrightarrow{KMnO_4}$ oxid. Ozonolysis

(2) B $CH_3-C\equiv N$ → $CH_3-C(CH_3)=O + NH_3$

CH_3MgBr

(3) D

Ph-NH → Ph-$\overset{+}{N}\equiv N$:

↓

Ph-CN → Ph-CH_2-NH_2 → (D)

$CH_3-C(CH_3)(CH_3)-OH$

(4) A ---- $6d^{10}\,7p^{6}\,8s^{2}$

112 118 120

117 ⇒ $7p^5$ ⇒ 17^{th} gp

120 ⇒ $8s^2$ ⇒ 2nd gp

(5) D

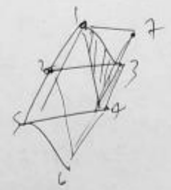
O=S=O

S ⇒ ... $3s^2 3p^4$

$3s^2$ $3p^{4-1}$ $3d^{0+1}$

pπ–pπ, pπ–dπ

(6) C Capped Octahedral



6 Ligands for Octahedral geometry and 7^{th} one makes trigonal pyramid with any one face.

sp^3d^3

$s + p_x + p_y + p_z + \{ d_{z^2}, d_{x^2-y^2}, d_{xy}$ or d_{z^2}, d_{zx}, d_{zy}

(7) B Na_2CO_3 stable

d-Block coloured ion

Inert gas → v.d. Waals Radius

Acidic oxide → SiO_2.

(8) A "CAR" concept !!!

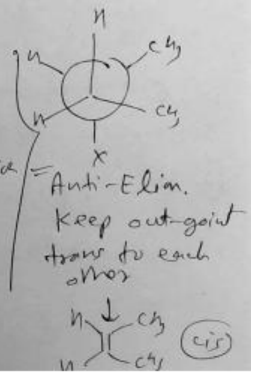
cis-Anti-Racemic

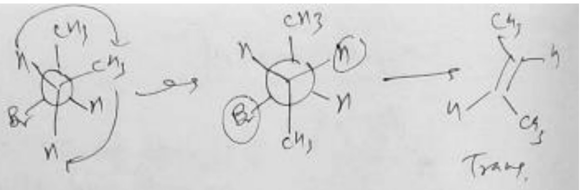
C	A	R
C	S	M
T	A	M
T	S	R

addition → ; Elimin. ←

$-HX \Rightarrow$ Anti-Elim.

Anti-Elim. Keep out-going trans to each other





(9) B (i) Aromatic halides ⇒ Least reactive for S_N1 & S_N2 both

(ii) for S_N2 $2° > 3°$

(iii) I, II ⇒ Leads, only in one option (B)

(10) B $S_N2 \Rightarrow$ Inversion

(11) A Ozonolysis, MHC !!! (Oxidative)

(12) D $[\overset{III}{Co}(NH_3)_5CO_3]ClO_4$

$x + 0 - 2 - 1 = 0$

$\therefore x = +3 \Rightarrow$ oxid. state.

Co(III) --- $4s^0 3d^6$, strong Ligand Field, d^2sp^3

(13) C cyclic phosphates, $(NPO_3)_n$

$N_5P_5O_{15}$

(14) B $PO_4^{3-} \Rightarrow$ mol. mass $31 + 64 = 95$

moles $= \frac{9.5}{95} = 0.1$ —(i)

$P \Rightarrow 15e^-$

$O_4 \Rightarrow 8 \times 4 = 32e^-$ } $\Rightarrow 50e^-$ —(ii)

$3- \Rightarrow 3e^-$

$\therefore 50 \times 0.1 = 5$ mole

(15) C $3p \Rightarrow RN = n - l - 1 = 3 - 1 - 1 = 1$

(16) B Anode (+) $H_2 \rightarrow 2H^+ + 2e^-$

Cathode (−) $2Ag^+ + 2e^- \rightarrow 2Ag$

(17) B All s^{-1} unit ⇒ Ist Order —(i)

$\frac{+d[P]}{dt} = r = k[A]^1$ —(ii)

rate = slope = tan 30°

$\sin 30° = \frac{1}{2}$ ⇒ $\tan 30° = \frac{1}{\sqrt{3}}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\therefore \frac{1}{\sqrt{3}} = k \times 0.1 \Rightarrow k = \frac{10}{\sqrt{3}} = \frac{10}{1.73} = 5.77 s^{-1}$

(18) C Monoatomic $C_V = \frac{3}{2}R$

$Q_{AB} = nC_V dT$

$= 1 \times \frac{3}{2}R(300) = 450R$

$Q_{CA} = nC_P dT = 1 \times \frac{5}{2}R(300 - 450)$

$= \frac{5}{2}R(-150) = -375R$

(19) B

$N_2O_4 \rightleftharpoons 2NO_2$

$1-\alpha$ $\quad 2\alpha$

$K_P = \frac{(2\alpha)^2}{(1-\alpha)}\left(\frac{P}{1+\alpha}\right)^1 = \frac{4\alpha^2 P}{1-\alpha^2}$

$\frac{1-\alpha^2}{\alpha^2} = \frac{4P}{K_P} \Rightarrow \frac{1}{\alpha^2} = \frac{4P}{K_P} + 1$

$\frac{1}{\alpha^2} = \frac{4P + K_P}{K_P}$

$\therefore \alpha = \sqrt{\frac{K_P}{4P + K_P}}$

(20) A

$\frac{P_A^0}{P_B^0} = \frac{1}{3}$ —(i)

$P_A = x_A P_A^0 = x'_A P_T$ —(ii)

$P_B = x_B P_B^0 = x'_B P_T$ —(iii)

(ii)/(iii) ⇒ $\frac{x_A}{x_B} \frac{P_A^0}{P_B^0} = \frac{x'_A}{x'_B}$

$\frac{x_A}{x_B} = \frac{4}{3} \times \frac{3}{1} = \frac{4}{1}$

$x_A = 4x_B = 1 - x_B \Rightarrow 5x_B = 1$

$\therefore x_B = \frac{1}{5}$

Ans (A)

(21) Ans. (2)

$RCH(COO^-)(NH_3^+) \xrightarrow{+H^+} RCH(COOH)NH_3$ — Cation form

AA

$\downarrow +OH^-$

$R-CH(NH_2)-COO^-$ — Anion Form

pH Range ⇒ 0 → (7) → 14 on OH^- Addition.

It means, initially, AA must be acidic. Hence, it should possess –COOH gp.

(22) Ans (2)

$ArCOOH \rightleftharpoons ArCOO^- + H^+ \quad K_a = 10^{-4}$

$ArCOO^-Na^+$ { Salt of strong base, weak Acid

	K_h	h	$[H^+]$	pH
(A) WA WB ⇒	$= \frac{K_w}{K_a \cdot K_b}$	$= \sqrt{\frac{K_w}{K_a \cdot K_b}}$	$\sqrt{\frac{K_w \cdot K_a}{K_b}}$	$\frac{1}{2}(14 + pK_a - pK_b)$

(B) WA SB $\quad [H^+] = \sqrt{\frac{K_w \cdot K_a}{C}} \Rightarrow pH = \frac{1}{2}(14 + pK_a + pC)$

$\therefore pH = 7 + \frac{1}{2}(4 + (-\log(1)))$

$= 7 + \frac{1}{2}(4 + (-2)) = 7 + \frac{1}{2}(2)$

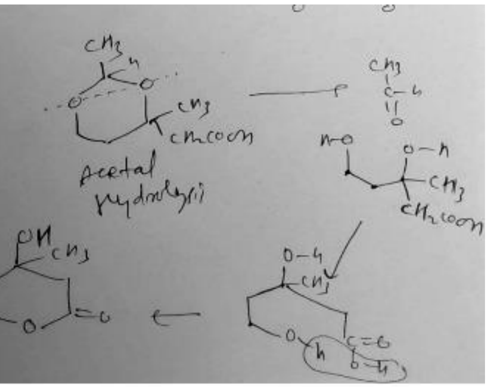
$= 8$ Ans.

(23) Ans (4) $\quad$ β-Keto-Acid or β-Acid-Acid

$R-\overset{O}{\overset{||}{C}}-CH_2-\overset{O}{\overset{||}{C}}-O-H$ (β, α)

$HO-\underset{O}{\underset{||}{C}}-CH_2-\underset{O}{\underset{||}{C}}-O-H$ (β, α)

(24) (3) Ans.



(25) Ans (9)

$\overset{+7}{KMnO_4} \xrightarrow{H^+} Mn^{+2}$

factor x = 5 $\quad N = M \times x$

1 ml, 1M ≡ 1 ml, 5N

$N_1V_1 = N_2V_2$

$5 \times 1 = 0.5 \times V_2$

$\therefore V_2 = \frac{5}{0.5} = 10$ ml.

$\therefore 1 + 9 \Rightarrow 10$

9 ml added.

Math

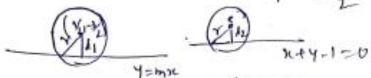
(1) Solution :-

$L_1 \Rightarrow y = mx \qquad L_2 \Rightarrow x + y = 1$

$x^2 + y^2 - x + 3y = 0$

According to question length of intercepts of a line is equal.

$C(1/2, -3/2) \quad r = \sqrt{\frac{1}{4} + \frac{9}{4}} = \frac{\sqrt{10}}{2}$



$2\sqrt{r^2 - l_1^2} = 2\sqrt{r^2 - l_2^2}$

$r^2 - l_1^2 = r^2 - l_2^2$

$l_1^2 = l_2^2 \Rightarrow l_1 = \pm l_2$

$\frac{\left|\frac{m}{2} + \frac{3}{2}\right|}{\sqrt{m^2+1}} = \pm \frac{\left|\frac{1}{2} - \frac{3}{2} - 1\right|}{\sqrt{2}}$

$\frac{|m+3|}{2\sqrt{m^2+1}} = \pm\sqrt{2}$

$(m+3)^2 = 8m^2 + 8$

$m^2 + 6m + 9 = 8m^2 + 8$

$7m^2 - 6m - 1 = 0$

$7m^2 - 7m + m - 1 = 0$

$7m(m-1) + 1(m-1) = 0$

$(7m+1)(m-1) = 0$

$m = -1/7, 1$

$y = -\frac{1}{7}x \qquad y = x$

$7y + x = 0, \quad y = x$

(2) $ax^2+bx+c=0$ $\quad px^2+qx+r=0$

α_1, α_2 $\quad \beta_1, \beta_2$

$\alpha_1 y+\alpha_2 z=0$ & $\beta_1 y+\beta_2 z=0$ has non trivial sdn.

then $\begin{vmatrix}\alpha_1 & \alpha_2\\ \beta_1 & \beta_2\end{vmatrix}=0$ $\quad \alpha_1\beta_2-\alpha_2\beta_1=0$

$\frac{\alpha_1}{\alpha_2}=\frac{\beta_1}{\beta_2} \Rightarrow \frac{\alpha_1-\alpha_2}{\alpha_1+\alpha_2}=\frac{\beta_1-\beta_2}{\beta_1+\beta_2}$

$\frac{(\alpha_1-\alpha_2)^2}{(\alpha_1+\alpha_2)^2}=\frac{(\beta_1-\beta_2)^2}{(\beta_1+\beta_2)^2}$

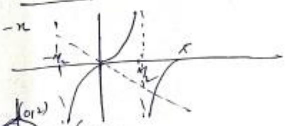
$\frac{b^2-4ac/a^2}{b^2/a^2}=\frac{q^2-4pr/p^2}{q^2/p^2}$

$\frac{b^2-4ac}{b^2}-1=\frac{q^2-4pr}{q^2}-1$

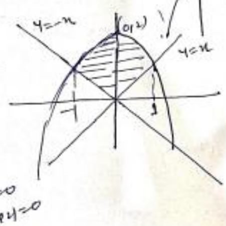
$\frac{4ac}{b^2}=\frac{4pr}{q^2} \Rightarrow \frac{b^2}{q^2}=\frac{ac}{pr}$

(3) by intermediate value theorem or

(3) $\tan x=-x$



(4) $y^2=x^2$



$y=2-x^2$

$y=2-y^2$

$y^2+y-2=0$

$y^2+2y-y-2=0$

$y(y+2)-1(y+2)=0$

$(y-1)(y+2)=0$

$y=1$ $\quad -2X$

$Ar=2\int_0^1(2-x^2-x)\,dx$

$Ar=2\left(2x-\frac{x^3}{3}-\frac{x^2}{2}\right)_0^1$

$=2\left(2-\frac{1}{3}-\frac{1}{2}\right)$

$=\frac{1}{3}(12-2-3)=7/3$

(5) $f:[0,16]\to R$ $\quad 0<\alpha<1,\ 1<\beta<2$

$g(x)=\int_0^{x^4}f(t)\,dt$ $\quad g(0)=0$

$g(2)=\int_0^{16}f(t)\,dt=I$

$I=\int_0^{16}f(t)\,dt=g(2)$

$=g(2)-g(1)+g(1)-g(0)$

$=\frac{g(2)-g(1)}{2-1}+\frac{g(1)-g(0)}{1}$

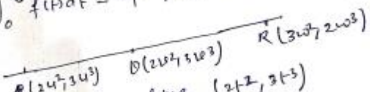
$=g'(\beta)+g'(\alpha)$ $\quad$ By L.M.V.T

$1<\beta<2$ & $0<\alpha<1$

$g'(x)=\int_0^{x^4}f(x^4)(4x^3)$

$I=\int_0^{16}f(t)\,dt=4\beta^3 f(\beta^4)+4\alpha^3 f(\alpha^4)$

(6)



let a point on the line $(2t^2, 3t^3)$

ut line $ax+by+c=0$

$a2t^2+b3t^3+c=0$

$3bt^3+2at^2+c=0$

u, v, w

$\Sigma uv=0$

(7) (c) $\cos(\cos^{-1}x)+\sin^{-1}\left(\sin\left(\frac{1+x^2}{2}\right)\right)=2\sec^{-1}(\sec x)$

Pls check the solution

$x\in[-1,1]$ $\quad x+\frac{1+x^2}{2}=2x$

$1+x^2\in[1,2]$ $\quad 2x+1+x^2=4x$

$\frac{1+x^2}{2}\in[1/2,1]$ $\quad (x-1)^2=0$ $\quad x=1=a$

$[100a]=10$

(8) $\int\frac{dx}{x+x^7}-px$ $\quad I=\int\frac{x^6+1}{x(1/x^6+1)}-\frac{1}{x^7+x}\,dx$

$=\log x-p(x)+C$

(9) (a) $\int_0^{3\pi/4}\left(\sin x+x\sin x+\cos x-x\cos x\right)dx$

$\int_0^{3\pi/4}\left((\sin x+\cos x)+x(\sin x-\cos x)\right)$

~~$\int_0^{3\pi/4}\sin x+\cos x$~~

$\int_0^{3\pi/4}(-\cos x+\sin x)_0^{3\pi/4}+\left(x(-\cos x-\sin x)\right)_0^{3\pi/4}$

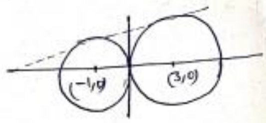
$+\int_0^{3\pi/4}(\cos x+\sin x)\,dx$

$\left(\left(\frac{1}{\sqrt2}+\frac{1}{\sqrt2}\right)-(-1)\right)+\left(\frac{3\pi}{4}\left(\frac{1}{\sqrt2}-\frac{1}{\sqrt2}\right)-0\right)$

$+(\sin x-\cos x)_0^{3\pi/4}$

$\sqrt2+1\ +\sqrt2+1=2(\sqrt2+1)$

(10)



$(x-3)^2+y^2=9$

$y=m(x-3)\pm3\sqrt{m^2+1}$

$(x+1)^2+y^2=1$ $\quad \sqrt{\frac{1}{3}+1}=\frac{2}{\sqrt3}$

$y=m(x+1)\pm\sqrt{m^2+1}$

$m\pm\sqrt{m^2+1}=-3m\pm3\sqrt{m^2+1}$

$m+\sqrt{m^2+1}=-3m+3\sqrt{m^2+1}$

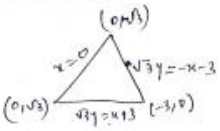
$2\cancel{4}m=2\sqrt{m^2+1}$

$4m^2=m^2+1$ $\quad 3m^2=1$

$m=\pm1/\sqrt3$

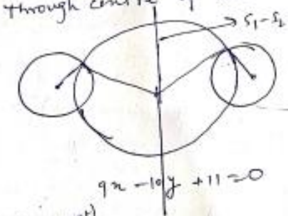
from $\sqrt3 y=x+3$

$\sqrt3 y=-x-3$



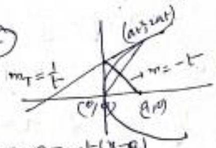
$$Ar = \frac{1}{2}\begin{vmatrix} 0 & -\sqrt{3} \\ 0 & \sqrt{3} \\ -3 & 0 \\ 0 & -\sqrt{3} \end{vmatrix} = \frac{1}{2}\left(3\sqrt{3} - (-3\sqrt{3})\right) = \frac{1}{2}\, 6\sqrt{3} = 3\sqrt{3}$$

(11) We know that if a variable circle intersect the two given circle orthogonally. In this condition radical axis of two circles passess through centre of variable circle.



$9x - 10y + 11 = 0$

(12)



$y - 0 = -t(x-a)$

$y = \frac{2at}{a+t^2}x$ $\qquad y = \frac{2}{t}\frac{2at}{2+t^2}$

$yt = 2x$ $\qquad y = \frac{2at}{2+t^2}$ —(2)

$(-tx + at)t = 2x$ $\qquad \frac{(1)}{(2)}\ \frac{x}{y} = \frac{t}{2}$

$-t^2x + at^2 = 2x$

$x = \frac{at^2}{2+t^2}$ —(1) $\qquad t = \frac{2x}{y}$

$x = a\,\frac{4x^2/y^2}{2 + \frac{4x^2}{y^2}}$

$2y^2 + 4x^2 = 4ax$

$\boxed{y^2 + 2x^2 = 2ax}$ ✓

(13) (a) $z + \frac{1}{z} = 2\cos\theta$

$z^2 - 2z\cos\theta + 1 = 0$

$z = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$

$z = \cos\theta \pm i\sin\theta$

$z = e^{i\theta}$ or $e^{-i\theta}$

$z^{2n} = e^{i2n\theta}$

$\left|\frac{z^{2n}-1}{z^{2n}+1}\right|$

$\left|\frac{\cos 2n\theta + i\sin 2n\theta - 1}{\cos 2n\theta + i\sin 2n\theta + 1}\right|$

$= \frac{\sqrt{(\cos 2n\theta - 1)^2 + \sin^2 2n\theta}}{\sqrt{(1+\cos 2n\theta)^2 + \sin^2 2n\theta}}$

$= \sqrt{\frac{1 + \cos^2 2n\theta - 2\cos 2n\theta + \sin^2 2n\theta}{1 + \cos^2 2n\theta + 2\cos 2n\theta + \sin^2 2n\theta}}$

$= \sqrt{\frac{2 \cdot 2\sin^2 n\theta}{2 \cdot 2\cos^2 n\theta}} = |\tan n\theta|$

(14) $x^3 - 3xy^2 + 2 = 0$ & $3x^2y - y^3 = 2$

$m_1 = \frac{dy}{dx}\Big|_{(x,y)} = +\left(\frac{3x^2 - 3y^2}{+6xy}\right)$ $\qquad m_2 = \frac{-6xy}{3x^2 - 3y^2}$

$\boxed{m_1 \times m_2 = -1}$ Angle b/w them is 90^0

(15) $\frac{\sin\theta + \sin 2\theta}{\cos\theta + \cos 2\theta} = \frac{2\sin\left(\frac{3\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2\cos\left(\frac{3\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}$

$= \tan\left(\frac{3\theta}{2}\right)$

period $= \frac{\pi}{\frac{3}{2}} = \frac{2\pi}{3}$

(16) We Know that

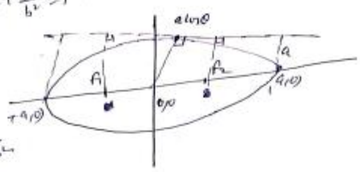
$$\frac{x^n - 1}{x-1} = (x-\alpha_1)(x-\alpha_2)\cdots(x-\alpha_{n-1})$$

Here

$$\lim_{x\to 1}\frac{x^n - 1}{x-1} = \lim_{x\to 1}(x-\omega)(x-\omega^2)\cdots(x-\omega^{n-1})$$

$$n = (1-\omega)(1-\omega^2)\cdots(1-\omega^{n-1})$$

(17) (d) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



$f_1 f_2 = b^2$

$y = mx \pm \sqrt{a^2m^2 + b^2}$

$mx - y \pm \sqrt{a^2m^2 + b^2} = 0$

$a = \frac{ma \pm \sqrt{a^2m^2 + b^2}}{\sqrt{m^2+1}}$ $\qquad a' = \frac{-ma \pm \sqrt{a^2m^2 + b^2}}{\sqrt{m^2+1}}$

$aa' = \left(\frac{\sqrt{a^2m^2+b^2} + ma}{\sqrt{m^2+1}}\right)\left(\frac{\sqrt{a^2m^2+b^2} - ma}{\sqrt{m^2+1}}\right)$

$aa' = \frac{a^2m^2 + b^2 - m^2a^2}{m^2+1} = \frac{b^2}{m^2+1}$

$c = \frac{\sqrt{a^2m^2+b^2}}{\sqrt{m^2+1}} \Rightarrow c^2 = \frac{a^2m^2+b^2}{m^2+1}$

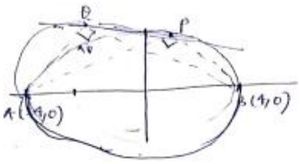
$\frac{b^2 - \frac{a^2m^2+b^2}{m^2+1}}{\frac{b^2}{m^2+1} - \frac{(a^2m^2+b^2)}{m^2+1}} = \frac{b^2m^2 + b^2 - a^2m^2 - b^2}{b^2 - a^2m^2 - b^2}$

$\frac{(b^2 - a^2)m^2}{-a^2m^2} = \frac{-a^2e^2}{-a^2} = e^2$

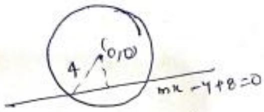
$b^2 = a^2 - a^2e^2$

(18) (d) do yourself

(19) (a)



$x^2 + y^2 = 16$



$\frac{8}{\sqrt{m^2+1}} < 4$

$m^2 + 1 > 4$

$m^2 > 3$

$m \in (-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

(20) (d)

(21) $x^2 - 2x\cos\theta + 1 = 0$

$x = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$

$x = \cos\theta \pm i\sin\theta = e^{i\theta}, e^{-i\theta}$

$x^{2n} = e^{i2n\theta}$

$(\cos 2n\theta + i\sin 2n\theta) - 2(\cos n\theta + i\sin n\theta)\cos n\theta + 1$

$\cos 2n\theta + i\sin 2n\theta - 2\cos^2 n\theta - i\,2\sin n\theta\cos n\theta + 1$

$2\cos^2 n\theta - 2\cos^2 n\theta = 0$

(22)

$\bar{A} = (2, 3, 6)$ $\bar{B} = (1, 1, -2)$ $\bar{C} = (1, 2, 1)$

$|\bar{A} \times (\bar{A} \times (\bar{A} \times \bar{B})) \cdot \bar{C}|$

$|\bar{A} \times ((\bar{A} \cdot \bar{B})\bar{A} - (\bar{A} \cdot \bar{A})\bar{B})\cdot \bar{C}|$

$|\{(\bar{A} \times \bar{A})(\bar{A} \cdot \bar{B}) - (\bar{A} \cdot \bar{A})(\bar{A} \times \bar{B})\} \cdot \bar{C}|$

$| |\bar{A}|^2 (\bar{A} \times \bar{B}) \cdot \bar{C}| = ||\bar{A}|^2 [\bar{A}\ \bar{B}\ \bar{C}]|$

$$[\bar{A}\ \bar{B}\ \bar{C}] = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 1 & -2 \\ 1 & 2 & 1 \end{vmatrix}$$

$= 2(1+4) - 3(1+2) + 6(2-1)$

$= 10 - 9 + 6 = 7$

$|A|^2 = 4 + 9 + 36 = 49$

$|A|^2 [\bar{A}\ \bar{B}\ \bar{C}] = 49 \times 7 = 343$

(23)

$T_n = \frac{1}{1+2+3+\dots n} = \frac{2}{n(n+1)}$

$S_n = \Sigma T_n = 2\left(1 - \frac{1}{n+1}\right)$

$n \to \infty$ $S_\infty = 2$

(24)

let $a_n = \int_0^{\pi/2} (1-\sin t)^n \sin 2t\, dt$

$1 - \sin t = y$

$-\cos t\, dt = dy$

$a_n = -\int_1^0 y^n\, 2(1-y)\, dy$

$= 2\int_0^1 (y^n - y^{n+1})\, dy$

$2\left(\frac{y^{n+1}}{n+1} - \frac{y^{n+2}}{n+2}\right)\Big|_0^1$

$a_n = 2\left(\frac{1}{n+1} - \frac{1}{n+2}\right)$

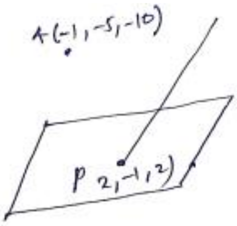
$= 2\,\frac{n+2-n-1}{(n+1)(n+2)} = \frac{2}{(n+1)(n+2)}$

$\lim_{n\to\infty} \sum_{n=1}^{n} \frac{2}{n(n+1)(n+2)}$

$\lim_{n\to\infty} \frac{2}{2}\left(\frac{1}{1\cdot 2} - \frac{1}{(n+1)(n+2)}\right)$

$= \frac{1}{2} = .5$

(25)



$P(3\lambda+2,\ 4\lambda-1,\ 12\lambda+2)$

$3\lambda + 2 - 4\lambda + 1 + 12\lambda + 2 = 5$

$11\lambda = 0$ $\lambda = 0$

$AP = \sqrt{9 + 16 + 144} = 13$